Previously:

Expressions

Values and **operations** that **evaluate** to a **value**

Values

- literals ✅
- variables ✅

Operators

- arithmetic ✅
- function calls ✅

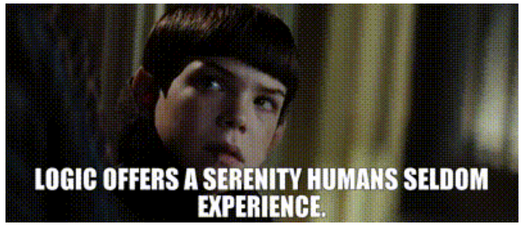
Today

Logic

Today:

Propositional logic (a.k.a., propositional calculus)

- propositions
- operators
- expressions



… as discrete math / philosophy

Don't let the word "calculus" scare you! There are many kinds of "calculus", and this one involves no differentiating or integrals.

If you go on to take a course in discrete mathematics (e.g., ECE 4110 or MATH 23020), you'll get into more advanced stuff like predicate logic with quantifiers. For our purposes, though, we will stick to nice, simple propositional logic.

Propositions

a.k.a., logical *statements*

P **:** it is sunny outside
Q **:** everyone in this room is taller than 6'
R **:** you passed the exam portion of Engineering 1020

Could be true or false!

Cannot be both true *and* false (Law of Non-Contradiction)
Must be either true *or* false (Law of Excluded Middle)

Operators

Apply to one or more propositions

Given propositions, can evaluate

Negation operator

¬ ("not")

P **:** it is sunny outside
$\neg P$ **:** it is not sunny outside

Unary **operator**

Q **:** there are more than 100 students here
$\neg Q$ **:** there are less than **or equal to** 100 students here

A *unary* operator operates on ____________ value... like a ________________ has ____________ wheel!

Remember: the opposite of $>$ isn't $<$, it's $\leq$!

Conjunction operator

$\wedge$ ("and")

Binary **operator**

P **:** it's over 20 degrees outside
Q **:** it's dry outside

Can you go out without a coat?

$$R = P \wedge Q$$

Truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

A binary operator has ______________________ in the same way that a ________________ has _____________ wheels

Disjunction operator

$\vee$ ("or")

P **:** transfer credit for MATH 1001
Q **:** passed 1001 with a grade ≥ 55

Have you satisfied the Eng One requirement for MATH 1001?

$$R = P \vee Q$$

Truth table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical *disjunction* is the *dual* of conjunction: whereas both operands to a conjunction ("and") operator have to be true to make the conjunction true, both operands to a disjunction ("or") have to be ____________ to make the disjunction ____________.

Exclusive disjunction

$\oplus$ ("xor")

Either this or that

P **:** switch at this end of the hall
Q **:** switch at the other end of the hall

Is the light on?

$$R = P \oplus Q$$

Truth table

P	Q	$P \vee Q$
T	T	F
T	F	T
F	T	T
F	F	F

The *exclusive or* operation turns out to be very important in computing. There are lots of things that can be either this or that, but ________________.

Truth table

Fill out as homework:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \oplus Q$
F	F				
F	T				
T	F				
T	T				

Operator properties

$\wedge$ is like multiplication

$\vee$ and $\oplus$ are like addition

Associative: $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

Commutative: $P \oplus Q \Leftrightarrow Q \oplus P$

Distributive: $P \wedge (Q \vee R) \Leftrightarrow P \wedge Q \vee P \wedge R$

The "and" and "or" operations behave like multiplication and addition in that the multiplicative operation ("and") comes before the additive ones ("or"), and also in that these operations share some common mathematical properties with their arithmetic cousins.

Useful expressions

Contradictions and tautologies tell us nothing:

$$R = P \wedge Q \wedge \neg P$$

$$R = (P \vee Q) \vee \neg(P \vee Q)$$

Contingencies **could be true or false, depending:**

$$R = \neg P \wedge Q$$

In order for a logical expression to be useful to us, it needs to be something that *could* evaluate to ________________________.

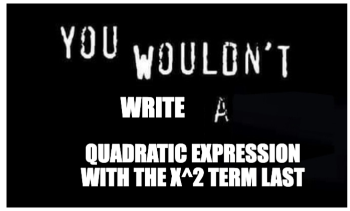
Imagine that I said I would give you a "pass" grade if your final exam mark was either ________________ or else ____________________. In that case, no matter how hard you work — or not! — it makes no difference to the outcome. That's not very motivational!

Normal form

What's wrong with this?

$$x + 9 + 3x^2$$

In one sense, nothing, but...



Similarly:

- $(P \lor \ldots) \land (Q \lor \ldots) \land \ldots$ (*conjunctive normal form*)
- $P \land \cdots \lor Q \land \cdots \lor \ldots$ (*disjunctive normal form*)

The *conjunctive normal form* should include only ANDs of terms that include only ORs. The *disjunctive normal form* should include only ORs of terms that include only ANDs. In both cases, negation should only be applied to specific propositions (e.g., $\neg P$), not expressions (e.g., $\neg(P \lor Q)$).

De Morgan's Laws

Expression negation

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

More complex example:

$$\neg(P \wedge \neg Q \vee R) = \neg((P \wedge \neg Q) \vee R)$$
$$= \neg(P \wedge \neg Q) \wedge \neg R = (\neg P \vee \neg\neg Q) \wedge \neg R$$
$$= (\neg P \vee Q) \wedge \neg R$$

It's often the case that we have a complex logical expression and we need to find its opposite.

Say that it's safe to land an aircraft if the landing lights are on, the gear is down and ATC has given permission to land. What's the opposite of that logical expression? If ____________ of those conditions are not met, ___________________________.

Summary

Propositions

Operators: $\wedge$, $\vee$ and $\oplus$

Expressions

- precedence
- normal forms
- De Morgan's Laws